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Influences of large deformation and rotary inertia on wave propagation in piezoelectric cylindrically laminated shells in thermal environment

K. Dong, X. Wang *

Department of Engineering Mechanics, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China

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Abstract

Influences of large deformation (geometrical non-linear) and rotary inertia on wave propagation in a long, piezoelectric cylindrically laminated shell in thermal environment is presented in this paper. Nonlinear dynamic governing equations of piezoelectric cylindrically laminated shells are derived by means of Hamilton's principle. The wave propagation modes are obtained by solving an eigenvalue problem. Numerical examples show that the characteristics of wave propagation in piezoelectric cylindrically laminated shells are relates to the large deformation, rotary inertia and thermal environment of the piezoelectric cylindrically laminated shells. The effect of large deformation, rotary inertia and thermal load on wave propagation in the piezoelectric cylindrically laminated shells is discussed by comparing with the result from the small deformation (geometrical linear shell theory). This method may be used to investigate wave propagation in various laminated material, layers numbers and thickness of piezoelectric cylindrically laminated shells under large deformation. The results carried out can be used in the ultrasonic inspection techniques and structural health monitoring.

Keywords: Piezoelectric cylindrically laminated shells; Wave propagation; Large deformations; Rotary inertia; Thermal environment

1. Introduction

The characteristic of wave propagation in elastic media can be used to predict the size of damage in a structure or used in the ultrasonic inspection techniques and structural health monitoring. There are many investigations of wave propagation in cylinder shell in the past decades: The membrane shell model was put

* Corresponding author. Fax: +86 21 6293 2045.
E-mail address: xwang@sjtu.edu.cn (X. Wang).

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forth by Love (1944), in which the transverse forces, bending and twisting moments are negligible, such model is suitable for thin shell structures in which only the normal and shear membrane forces acting in the mid-surface of shell are considered. For shells of moderate thickness, Mirsky and Herrmann (1957) studied the shear effects in both axial and circumferential directions, and the rotary-inertia effects in the study of axially symmetric waves in a cylindrical shell; Lin and Morgan (1956) developed the equations for axially symmetric motions including both shear and rotary inertia effects for non-axially symmetric motion of shell structures. Cooper and Naghdi (1957) presented a theory including shear and rotary inertia effects for non-axially symmetric motion of shell structures. Mirsky (1964) studied an approximate theory for vibration of orthotropic thick cylindrical shell in which the effect of transverse normal stress was retained.

The other hands, there are also have many researches about the large deformation (geometrical nonlinear) theory of shells. Donnell (1934) derived governing equations of large deformations cylindrical shells based on some approximate hypothesis. Von Karman and Tsion (1939, 1941) studied the buckling and postbuckling of thin cylindrical spherical shells by using large deformation theory of shells. Reissner (1977) discussed small deflection (geometrical linear) theory of laminated shell considering the effect of normal direction strain. Wampner (1967) derived a theory for moderately large deflections of sandwich shell with dissimilar facing.

In recent years, the use of piezoelectric materials in intelligent structures attracted extensive attentions. Due to the intrinsic direct and converse piezoelectric effects, piezoelectric materials can be effectively used as sensors or actuators for the active shape or vibration control of structures (Li and Lin, 2001). One of the important applications of wave propagation in piezoelectric structures is the using of interdigital transducer (IDT). IDT was first used to excite the surface wave devices in radar communication equipment as filters and delay lines (Varadan and Varadan, 2000), so it is important to know the characteristic of wave propagation in piezoelectric structures. Wave propagation and vibration in pure piezoelectric structures and laminated elasitic structures have attracted considerable attention of many researchers (Wang et al., 2002; Wang and Dai, 2004a,b; Dai and Wang, 2004, 2005). But the study of wave propagation in laminated piezoelectric cylinder shells is only in recent years: Wang (2002); Wang (2003a,b) discussed wave propagation in piezoelectric coupled cylinder affected by transverse shear and rotary inertia with Cooper–Naghdi shell theory. There are many investigations on the thermal load effect on piezoelectric cylindrical shell such as the constitutive relationship of piezoelectric materials thinking of electro-thermo-elastic properties is investigated by Tan and Tong (2002); A postbucking analysis is presented for a cross-ply laminated cylindrical shell with piezoelectric actuators subjected to the combined action of mechanical, electric and thermal load by Shen (2002); Wang (2003a,b) presented an analytical solution for the axisymmetric deformation of a finitely long and laminated cylindrical shell under pressuring loading and a uniform temperature change. Most of these studies are based on geometrically linear theory or static problems. Because many piezoelectric structures like lightweight space structures or thin-wall vessels are often used in thermal environments, and induced to a large deformation of the structures under large external static or dynamic loads, researches on the effects of large deformation on dynamic characteristics of structures are necessary in order to design and control the structural systems effectively. Baumhauer and Tiersten (1973) developed general piezoelectric nonlinear theory. Tzou and Bao (1993, 1997) presented a geometrical nonlinear theory of a piezoelectric laminated shell. Authors (Huang and Shen, 2004; Huang et al., 2004; Shen et al., 2003, 2004; Yang and Shen, 2003) utilized analytical methods based on plate and shell theories to solve nonlinear vibration and dynamic response of functionally graded plates in thermal environments and to study bending and vibration characteristics of damaged RC slabs strengthened with externally bonded CFRP sheets. However, to the authors' knowledge, there are a few investigations on the effects of large deformation on wave propagation in piezoelectric cylindrically laminated shells in thermal environment.

This paper presents an analytical method to study the effects of large deformation and rotary inertia on wave propagation in a long, piezoelectric cylindrically laminated shell with piezoelectric actuator layer and sensor layer in thermal environment. Based on Hamilton's principle, the nonlinear dynamic governing equations of piezoelectric cylindrically laminated shells are derived. The wave phase velocity is obtained by solving an eigenvalue problem. Numerical examples show that the characteristics of wave propagation in piezoelectric cylindrically laminated shells are relates to the large deformation of the piezoelectric cylindrically laminated shells, rotary inertia and thermal environment of the piezoelectric cylindrically laminated shells. The effects of large deformation, rotary inertia and thermal load on wave propagation in the piezoelectric cylindrically laminated shells are discussed by comparing with the result from the small deformation (geometrical linear shell theory). In order to prove the validity of our solution method and numerical results further, wave propagation in piezoelectric cylindrically laminated shells under small deformation from the present method omitting geometrical non-linear term in geometrical equations is nearly agreement with some previous results based on a different analytical method in the literature (Wang, 2003a,b). Thus, this method may be used to investigate wave propagation in various laminated material, layers numbers and thickness of piezoelectric cylindrically laminated shells under large deformation and thermal environment. Results carried out can be used in the ultrasonic inspection techniques and structural health monitoring.

2. Non-linear governing equation

A long, piezoelectric cylindrically laminated shell with (1) orthotropic elasticity host layer, (2) piezoelectric actuator layer and (3) piezoelectric sensor layer is shown in Fig. 1. The curvilinear coordinate system (x_1, x_2, x_3) is shown in Fig. 1, where x_1 expresses the axial coordinate, x_2 expresses the circumferential coordinate, and x_3 expresses the radial coordinate. The surface defined by $x_3 = 0$ expresses the middle surface of the laminated shells, and R is the radius of piezoelectric cylindrically laminated shells.

2.1. Constitutive relations

The material properties are assumed to be independent of temperature, and the stress and strain relations are linear. The constitutive relations of orthotropic piezoelectric materials in thermal environment are written as (Tiersten, 1969)

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{33} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} - \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Theta, \tag{1}$$

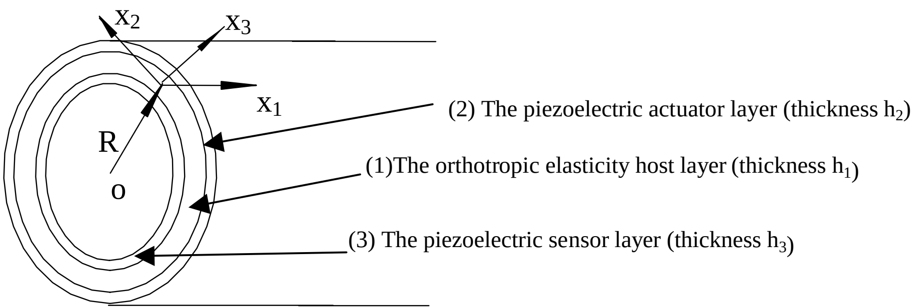

Fig. 1. Cylindrical shells coated with piezoelectric layers.

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} + \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} - \begin{Bmatrix} p_1 \\ p_2 \\ p_3 \end{Bmatrix} \Theta, \tag{2}$$

where σ_i, S_i, D_i, E_i $(i = 1, 2, 3, \ldots, 6)$ represent the stresses strains, electric displacements, electric fields, respectively; c_{ij}, e_{ij}, g_{ij} denote the elastic constants, piezoelectric constants and dielectric constants, respectively; λ_i and $p_i (i = 1, 2, 3)$ are the thermal modulus and pyroelectric constants respectively; Θ expresses temperature change in the piezoelectric cylindrically laminated shells.

The relations between the electric fields$E_i (i = 1, 2, 3)$ and the electric potential ϕ in the curvilinear coordinate system are defined by

$$E_1 = -\frac{\partial \phi}{\partial x_1}, \quad E_2 = -\frac{1}{R + x_3} \cdot \frac{\partial \phi}{\partial x_2}, \quad E_3 = -\frac{\partial \phi}{\partial x_3}. \tag{3}$$

2.2. Non-linear geometrical relations

The non-linear strain of piezoelectric cylindrically laminated shells can be introduced by a large deformation which is induced by mechanical and/or electric loads. According to the Love–Kirchhoff thin shell assumptions, the nonlinear displacements $U_i(x_1, x_2, x_3, t)(i = 1, 2, 3)$ in the i th direction can be expressed as (Tzou et al., 1993)

$$U_1(x_1, x_2, x_3, t) = u_1(x_1, x_2, t) + \theta_1(x_1, x_1, t) \cdot x_3 \tag{4a}$$

$$U_2(x_1, x_2, x_3, t) = u_2(x_1, x_2, t) + \theta_2(x_1, x_1, t) \cdot x_3 \tag{4b}$$

$$U_3(x_1, x_2, x_3, t) = u_3(x_1, x_2, t), \tag{4c}$$

where $u_i(x_1, x_2, x_3, t)(i = 1, 2, 3)$ is the displacement component of a point on the mid-plane of the shell along the $x_i (i = 1, 2, 3)$ axis; θ_1 and θ_2 represent the rotations of a transverse normal at $x_3 = 0$ around the x_2 and x_1 axis, respectively. The laminated shell is considered to be thin, so that the transverse normal strains S_3 and shear strains S_4, S_5 are negligible. Thus, the rotational angles θ_1 and θ_2 for the thin laminated shell are defined as (Tzou and Bao, 1997)

$$\theta_1 = -\frac{\partial u_3}{\partial x_1}, \theta_2 = \frac{u_2}{R} - \frac{1}{R} \frac{\partial u_3}{\partial x_2}. \tag{5}$$

In general, for a thin shell the in-plane displacements are much smaller than the transverse deflection. Thus, the nonlinear effects due to the in-plane deformation are usually neglected (Palazatto and Dennis, 1992), only the nonlinear strains due to the large transverse deflection u_3 are considered. Therefore, the nonlinear geometrical relations can be written as

$$\begin{Bmatrix} S_1 \\ S_2 \\ S_6 \end{Bmatrix} = \begin{Bmatrix} s_1 \\ s_2 \\ s_6 \end{Bmatrix} + x_3 \cdot \begin{Bmatrix} k_1 \\ k_2 \\ k_6 \end{Bmatrix} \tag{6}$$

where s_1, s_2, s_6 are the membrane strains in the plane (x_1–x_2), and k_1, k_2, k_6 are the bending curvatures. Subscripts 1, 2, 6 denote the two normal strains and a shear strain in the plane (x_1–x_2), respectively. The nonlinear strains are written as functions of displacements u_i $(i = 1, 2, 3)$ of the mid-plane as follows:

$$s_1 = \frac{\partial u_1}{\partial x_1} + \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1}\right)^2 \tag{7a}$$

$$s_2 = \frac{1}{R}\cdot\frac{\partial u_2}{\partial x_2} + \frac{u_3}{R} + \frac{1}{2}\cdot\frac{1}{R^2}\left(\frac{\partial u_3}{\partial x_2}\right)^2 \tag{7b}$$

$$s_6 = \frac{1}{R}\cdot\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} + \frac{1}{R}\cdot\frac{\partial u_3}{\partial x_1} \tag{7c}$$

$$k_1 = -\frac{\partial^2 u_3}{\partial x_1^2}, \quad k_2 = \frac{\partial u_2}{R^2 \partial x_2} - \frac{1}{R^2}\cdot\frac{\partial^2 u_3}{\partial x_2^2}, \quad k_6 = \frac{\partial u_2}{R\partial x_1} - \frac{2}{R}\cdot\frac{\partial^2 u_3}{\partial x_1 \partial x_2}. \tag{7d}$$

2.3. Hamilton's principle and nonlinear governing equations

The nonlinear dynamic equations of piezoelectric cylindrically laminated shells can be derived by utilizing Hamilton's principle as follows:

$$\delta \int_{t_0}^{t_1} (T - \Pi)\,\mathrm{d}t = 0, \tag{8}$$

where

$$T = \int_V \frac{1}{2}\rho \dot{U}_j \dot{U}_j \,\mathrm{d}V \tag{8a}$$

is the kinetic energy,

$$\Pi = \int_v \hbar(S_i, E_j)\,\mathrm{d}V \tag{8b}$$

is the total potential energy, and

$$\hbar = \{S\}^{\mathrm{T}}[c]\{S\}/2 - \{E\}^{\mathrm{T}}[g]\{E\}/2 - \{E\}^{\mathrm{T}}[g]/2 - \{E\}^{\mathrm{T}}[e]\{S\} \tag{8c}$$

is the electric enthalpy. In the above formula, ρ is the mass density; U_j and $\dot{U}_j$ are the displacement and velocity, respectively; and V is the volume around piezoelectric cylindrically laminated shells.

Here, a piezoelectric cylindrically laminated shell is considered as a symmetrical laminated structure (the thickness of two piezoelectric layers is identical, $h_2 = h_3$, or the thickness of two piezoelectric layers is much less than the thickness of the elastic host layer, $h_2, h_3 \ll h_1$). Thus, it is reasonable that the middle surface of the elastic host layer is taken as the neutral plane of the piezoelectric cylindrically laminated shells. Substituting Eq. (4) into Eq. (1) and integrating the stresses across the thickness of the piezoelectric cylindrically laminated shells, give the membrane stresses and bending stresses as

$$N_1 = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} \sigma_1^1\,\mathrm{d}x_3 + \int_{\frac{h_1}{2}}^{\frac{h_1}{2}+h_2} \sigma_1^2\,\mathrm{d}x_3 + \int_{-\frac{h_1}{2}-h_3}^{-\frac{h_1}{2}} \sigma_1^3\,\mathrm{d}x_3 = A_1 s_1 + A_2 s_2 + A_3 k_1 + A_4 k_2 + A_5 \frac{\partial \varphi}{\partial x_1} + A_6 \Theta, \tag{9}$$

$$N_2 = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} \sigma_2^1\,\mathrm{d}x_3 + \int_{\frac{h_1}{2}}^{\frac{h_1}{2}+h_2} \sigma_2^2\,\mathrm{d}x_3 + \int_{-\frac{h_1}{2}-h_3}^{-\frac{h_1}{2}} \sigma_2^3\,\mathrm{d}x_3 = B_1 s_1 + B_2 s_2 + B_3 k_1 + B_4 k_2 + B_5 \frac{\partial \varphi}{\partial x_1} + B_6 \Theta, \tag{10}$$

$$N_6 = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} \sigma_6^1\,\mathrm{d}x_3 + \int_{\frac{h_1}{2}}^{\frac{h_1}{2}+h_2} \sigma_6^2\,\mathrm{d}x_3 + \int_{-\frac{h_1}{2}-h_3}^{-\frac{h_1}{2}} \sigma_6^3\,\mathrm{d}x_3 = C_1 s_6 + C_2 k_6 + C_3 \frac{\partial \varphi}{R \partial x_2}, \tag{11}$$

$$M_1 = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} x_3 \cdot \sigma_1^1 \, dx_3 + \int_{\frac{h_1}{2}}^{\frac{h_1}{2}+h_2} x_3 \cdot \sigma_1^2 \, dx_3 + \int_{-\frac{h_1}{2}-h_3}^{-\frac{h_1}{2}} x_3 \cdot \sigma_1^3 \, dx_3,$$
$$= D_1 s_1 + D_2 s_2 + D_3 k_1 + D_4 k_2 + D_5 \frac{\partial \varphi}{\partial x_1} + D_6 \Theta, \tag{12}$$
$$M_2 = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} x_3 \cdot \sigma_2^1 \, dx_3 + \int_{\frac{h_1}{2}}^{\frac{h_1}{2}+h_2} x_3 \cdot \sigma_2^2 \, dx_3 + \int_{-\frac{h_1}{2}-h_3}^{-\frac{h_1}{2}} x_3 \cdot \sigma_2^3 \, dx_3,$$
$$= E_1 s_1 + E_2 s_2 + E_3 k_1 + E_4 k_2 + E_5 \frac{\partial \varphi}{\partial x_1} + E_6 \Theta, \tag{13}$$
$$M_6 = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} x_3 \cdot \sigma_6^1 \, dx_3 + \int_{\frac{h_1}{2}}^{\frac{h_1}{2}+h_2} x_3 \cdot \sigma_6^2 \, dx_3 + \int_{-\frac{h_1}{2}-h_3}^{-\frac{h_1}{2}} x_3 \cdot \sigma_6^3 \, dx_3 = F_1 s_6 + F_2 k_6 + F_3 \frac{\partial \varphi}{R \partial \theta}, \tag{14}$$

where N_1, N_2 and N_6 are the membrane forces, M_1, M_2 and M_6 are the internal moment, σ_i^j represent the stresses (subscripts $i = 1, 2, 6$ express the normal stress and shear stress in the plane (1, 2), and superscripts $j = 1, \ldots, 3$ express the elasticity layer, actuator layer and sensor layer, respective). The expressions of A_i, B_i, D_i, E_i ($i = 1, \ldots, 6$)and C_i, F_i ($i = 1, \ldots, 3$) are shown in Appendix A.

From Eq. 4, the inertia terms can be written as

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_k \ddot{U}_j \, dx_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_k (\ddot{u}_j + x_3 \ddot{\theta}_j) \, dx_3 = \rho_s h \ddot{u}_j, \quad (j = 1, 2), \tag{15a}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_k \ddot{U}_j x_3 \, dx_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_k (\ddot{u}_j + x_3 \ddot{\theta}_j) x_3 \, dx_3 = I \ddot{\theta}_j, \quad (j = 1, 2), \tag{15b}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_k \ddot{U}_3 \, dx_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_k \ddot{u}_3 \, dx_3 = \rho_s h \ddot{u}_3, \tag{15c}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_k \ddot{U}_j x_3^2 \, dx_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_k (\ddot{u}_j + x_3 \ddot{\theta}_j) x_3^2 \, dx_3 = J \ddot{u}_j \quad (j = 1, 2), \tag{15d}$$

where $\rho_s = (\sum_{k=1}^{3} \rho_k h_k)/h$ is defined as a weight average density for the piezoelectric laminated shells, $I\ddot{\theta}_j$ and $J\ddot{u}_j$ are the rotary inertia terms as follows:

$$I = J = \frac{\rho_1 h_1^3}{12} + \frac{\rho_2}{3} \left(h_2^3 + \frac{3}{2} h_1 h_2^2 + \frac{3}{4} h_1^2 h_2 \right) + \frac{\rho_3}{3} \left(h_3^3 + \frac{3}{2} h_1 h_3^2 + \frac{3}{4} h_1^2 h_3 \right). \tag{15e}$$

Utilizing the Hamilton's equation (8) and considering the effects of piezoelectric layers, the non-linear dynamic governing equations of piezoelectric cylindrically laminated shells in thermal environment are given by

$$\frac{\partial N_1}{\partial x_1} + \frac{\partial N_6}{R \partial x_2} = \rho_s h \frac{\partial^2 u_1}{\partial t^2} + \frac{I}{R} \frac{\partial^2 \theta_1}{\partial t^2}, \tag{16}$$

$$\frac{\partial N_2}{R \partial x_2} + \frac{\partial N_6}{\partial x_1} + \frac{\partial M_2}{R^2 \partial x_2} + \frac{\partial M_6}{R \partial x_1} = \rho_s h \frac{\partial^2 u_2}{\partial t^2} + 2 \cdot \frac{I}{R} \frac{\partial^2 \theta_2}{\partial t^2} + \frac{J}{R^2} \frac{\partial^2 u_2}{\partial t^2}, \tag{17}$$

$$\frac{\partial^2 M_1}{\partial x_1^2} + \frac{\partial^2 M_6}{R \partial x_1 \partial x_2} + \frac{1}{R} \cdot \left(\frac{\partial^2 M_2}{R \partial x_2^2} + \frac{\partial^2 M_6}{\partial x_1 \partial x_2} \right) - \frac{N_2}{R} + \left[\frac{\partial}{\partial x_1} \left(N_1 \frac{\partial u_3}{\partial x_1} \right) \right.$$

$$+\frac{1}{R^2}\cdot\frac{\partial}{\partial x_2}\left(\frac{\partial u_3}{\partial x_2}\right)+\frac{1}{R}\frac{\partial N_6}{\partial x_1}\frac{\partial u_3}{\partial x_2}+\frac{1}{R}\frac{\partial N_6}{\partial x_2}\frac{\partial u_3}{\partial x_1}+\frac{2}{R}N_6\frac{\partial^2 u_3}{\partial x_1\partial x_2}\Bigg]$$
$$=\rho_s h\frac{\partial^2 u_3}{\partial t^2}+I\frac{\partial^3\theta_1}{\partial x_1\partial t^2}+J\frac{\partial^3 u_1}{R\partial x_1\partial t^2}+\frac{I}{R}\frac{\partial^3\theta_2}{\partial x_2\partial t^2}+\frac{J}{R^2}\left(\frac{\partial^3 u_2}{\partial x_2\partial t^2}\right). \tag{18}$$

3. Dispersion characteristics

Substituting Eqs. (9)–(14) into Eqs. (16)–(18), yields

$$A_1\frac{\partial^2 u_1}{\partial x_1^2}+\frac{A_2}{R}\left(\frac{\partial^2 u_2}{\partial x_1\partial x_2}+\frac{\partial u_3}{\partial x_1}\right)-A_3\cdot\frac{\partial^3 u_3}{\partial x_1^3}+\frac{A_4}{R^2}\left(\frac{\partial^2 u_2}{\partial x_1\partial x_2}-\frac{\partial^3 u_3}{\partial x_1\partial x_2^2}\right)+A_5\frac{\partial^2\varphi}{\partial x_1^2}+\frac{C_1}{R^2}\frac{\partial^2 u_1}{\partial x_2^2}$$
$$+\frac{C_1}{R}\frac{\partial^2 u_2}{\partial x_1\partial x_2}+\frac{C_2}{R^2}\left(\frac{\partial^2 u_2}{\partial x_1\partial x_2}-2\frac{\partial^3 u_3}{\partial x_1\partial x_2^2}\right)+\frac{C_3}{R^2}\frac{\partial^2\varphi}{\partial x_2^2}+A_6\frac{\partial\Theta}{\partial x_1}=\rho_S\frac{\partial^2 u_1}{\partial t^2}-\frac{I}{R}\frac{\partial^3 u_3}{\partial x_1\partial t^2}, \tag{19}$$

$$\left(\frac{B_1}{R}+\frac{C_1}{R}+\frac{E_1}{R^2}+\frac{F_1}{R^2}\right)\cdot\frac{\partial^2 u_1}{\partial x_1\partial x_2}+\left(\frac{B_2}{R^2}+\frac{B_4}{R^3}+\frac{E_2}{R^3}+\frac{E_4}{R^4}\right)\cdot\frac{\partial^2 u_2}{\partial x_2^2}+\left(\frac{B_2}{R}+\frac{E_2}{R^3}\right)\cdot\frac{\partial u_3}{\partial x_2}$$
$$-\left(\frac{B_3}{R}+\frac{2C_2}{R}+\frac{2F_2}{R^2}+\frac{E_3}{R^2}\right)\cdot\frac{\partial^3 u_3}{\partial x_1^2\partial x_2}-\left(\frac{B_4}{R^3}+\frac{E_4}{R^4}\right)\cdot\frac{\partial^3 u_3}{\partial x_2^3}+\left(C_1+\frac{C_2}{R}+\frac{F_1}{R}+\frac{F_2}{R^2}\right)\cdot\frac{\partial^2 u_2}{\partial x_1^2}$$
$$+\left(\frac{B_5}{R}+\frac{C_3}{R}+\frac{E_5}{R^2}+\frac{F_3}{R^2}\right)\cdot\frac{\partial^2\varphi}{\partial x_1\partial x_2}+\left(\frac{B_6}{R}+\frac{E_6}{R^2}\right)\frac{\partial\Theta}{\partial x_2}=\rho_s\frac{\partial^2 u_2}{\partial t^2}+\frac{I}{R^2}\left(3\frac{\partial^2 u_2}{\partial t^2}-2\frac{\partial^3 u_3}{\partial x_2\partial t^2}\right), \tag{20}$$

$$D_1\frac{\partial^3 u_1}{\partial x_1^3}+\left(\frac{D_2}{R}+\frac{D_4}{R^2}+\frac{F_1}{R}+\frac{F_2}{R^2}+\frac{F_1}{R}+\frac{F_2}{R^2}\right)\cdot\frac{\partial^3 u_2}{\partial x_1^2\partial x_2}+\left(\frac{D_2}{R}-\frac{B_3}{R}\right)\cdot\frac{\partial^2 u_3}{\partial x_1^2}-D_3\frac{\partial^4 u_3}{\partial x_1^4}$$
$$+\left(-\frac{D_4}{R^2}-\frac{2F_2}{R^2}-\frac{E_3}{R^2}-\frac{2F_2}{R^2}\right)\cdot\frac{\partial^4 u_3}{\partial x_1^2\partial x_2^2}+D_5\frac{\partial^3\varphi}{\partial x_1^3}+\left(\frac{F_1}{R^2}+\frac{E_1}{R^2}+\frac{F_1}{R^2}\right)\cdot\frac{\partial^3 u_1}{\partial x_1\partial x_2^2}$$
$$+\frac{F_3}{R^2}\frac{\partial^3\varphi}{\partial x_1\partial x_2^2}+\left(\frac{E_2}{R^3}+\frac{E_4}{R^4}\right)\cdot\frac{\partial^3 u_2}{\partial x_2^3}+\left(\frac{E_2}{R^3}+\frac{B_4}{R^3}\right)\cdot\frac{\partial^2 u_3}{\partial x_2^2}-\frac{E_4}{R^4}\frac{\partial^4 u_3}{\partial x_2^4}+\left(\frac{E_5}{R^2}+\frac{F_3}{R^2}\right)\cdot\frac{\partial^3\varphi}{\partial x_1\partial x_2^2}$$
$$-\frac{B_1}{R}\frac{\partial u_1}{\partial x_1}+\left(-\frac{B_2}{R^2}-\frac{B_4}{R^3}\right)\cdot\frac{\partial u_2}{\partial x_2}-\frac{B_2}{R^2}u_3-\frac{B_5}{R}\frac{\partial\varphi}{\partial x_1}+D_6\frac{\partial^2\Theta}{\partial x_1^2}+\frac{E_6}{R^4}\frac{\partial^2\Theta}{\partial x_2^2}$$
$$-\frac{B_6}{R}\Theta=\rho_s\cdot\frac{\partial^2 u_3}{\partial t^2}-I\left(\frac{\partial^4 u_3}{\partial x_1^2\partial t^2}-\frac{\partial^3 u_1}{R\partial x_1\partial t^2}\right)+\frac{I}{R^2}\left(2\frac{\partial^3 u_2}{\partial x_2\partial t^2}-\frac{\partial^4 u_3}{\partial x_2^2\partial t^2}\right). \tag{21}$$

Because the piezoelectric actuator layer and the piezoelectric sensor layer should, respectively, meet Maxwell equation $\int\nabla D\,dz=0$, we have

$$-g_{33p}^2h_2\cdot\frac{\partial^2\varphi}{\partial x_1^2}+e_{33p}^2h_2\cdot\frac{\partial^2 u_1}{\partial x_1^2}-\frac{1}{2}\cdot e_{33p}^2\left(h_1h_2+h_2^2\right)\cdot\frac{\partial^3 u_3}{\partial x_1^3}+\frac{e_{31p}^2}{R}h_2\cdot\left(\frac{\partial u_3}{\partial x_1}+\frac{\partial^2 u_2}{\partial x_1\partial x_2}\right)$$
$$+\frac{e_{31p}^2}{2R^2}\left(h_1h_2+h_2^2\right)\cdot\left(\frac{\partial^2 u_2}{\partial x_1\partial x_2}-\frac{\partial^3 u_3}{\partial x_1\partial x_2^2}\right)-g_{11p}^2h_2\frac{\partial^2\varphi}{R^2\partial x_2^2}+\frac{e_{15p}^2}{R^2}h_2\cdot\frac{\partial^2 u_1}{\partial x_2^2}$$
$$+\frac{e_{15p}^2}{R}h_2\cdot\frac{\partial^2 u_2}{\partial x_1\partial x_2}+\frac{e_{15p}^2}{2R^2}\left(h_1h_2+h_2^2\right)\cdot\left(\frac{\partial^2 u_2}{\partial x_1\partial x_2}-2\frac{\partial^3 u_3}{\partial x_1\partial x_2^2}\right)+p_3^2\frac{\partial\Theta}{\partial x_1}h_2=0, \tag{22}$$

$$- g_{33p}^3 h_3 \cdot \frac{\partial^2 \varphi}{\partial x_1^2} + e_{33p}^3 h_3 \cdot \frac{\partial^2 u_1}{\partial x_1^2} + \frac{1}{2} e_{33p}^3 (h_1 h_3 + h_3^2) \cdot \frac{\partial^3 u_3}{\partial x_1^3} + \frac{e_{31p}^3}{R} h_3 \cdot \left(\frac{\partial u_3}{\partial x_1} + \frac{\partial^2 u_2}{\partial x_1 \partial x_2} \right)$$
$$- \frac{e_{31p}^3}{2R^2} (h_1 h_3 + h_3^2) \cdot \left(\frac{\partial^2 u_2}{\partial x_1 \partial x_2} - \frac{\partial^3 u_3}{\partial x_1 \partial x_2^2} \right) - g_{11p}^3 h_3 \cdot \frac{\partial^2 \varphi}{R^2 \partial x_2^2} + \frac{e_{15p}^2}{R^2} h_3 \cdot \frac{\partial^2 u_1}{\partial x_2^2}$$
$$+ \frac{e_{15p}^3}{R} h_3 \cdot \frac{\partial^2 u_2}{\partial x_1 \partial x_2} - \frac{e_{15p}^3}{2R^2} (h_1 h_3 + h_3^2) \cdot \left(\frac{\partial^2 u_2}{\partial x_1 \partial x_2} - 2 \frac{\partial^3 u_3}{\partial x_1 \partial x_2^2} \right) + p_3^3 \frac{\partial \Theta}{\partial x_1} h_3 = 0. \tag{23}$$

Wave propagation modes in piezoelectric cylindrical laminated shells in thermal environment is described by

$$u_1(x_1, x_2, t) = U e^{i\xi(x_1 - ct)} \cos n x_2, \tag{24a}$$
$$u_2(x_1, x_2, t) = V e^{i\xi(x_1 - ct)} \sin n x_2, \tag{24b}$$
$$u_3(x_1, x_2, t) = W e^{i\xi(x_1 - ct)} \cos n x_2, \tag{24c}$$
$$\varphi(x_1, x_2, t) = \phi e^{i\xi(x_1 - ct)} \cos n x_2, \tag{24d}$$
$$\Theta(x_1, x_2, t) = T e^{i\xi(x_1 - ct)} \cos n x_2, \tag{24e}$$

where ξ and c are the wave number and wave phase velocity, respectively, and $\omega = \xi c$ is the eigen-frequency. Substituting Eq. (24) into Eqs. (19)–(21) and Eqs. (22) and (23), yields a set of homogeneous equations in terms of U, V, W, ϕ, T as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{Bmatrix} U \\ V \\ W \\ \phi \\ T \end{Bmatrix} = 0, \tag{25}$$

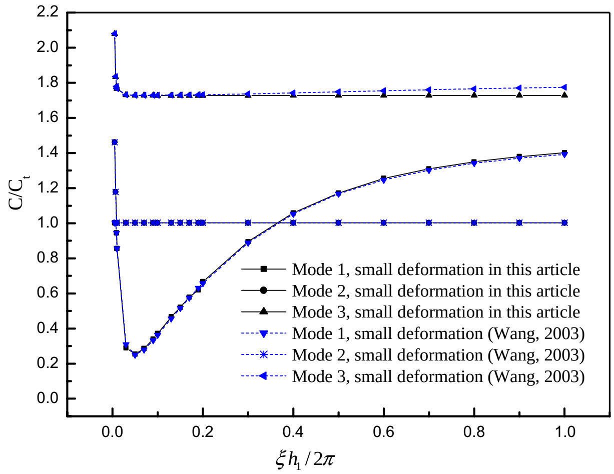

Fig. 2. Comparison of wave characteristics in piezoelectric cylindrically shells composed of a piezoelectric layer and a elastic host layer under small deformation from two different solution methods, where $h_1 R = 1/30$ and $h_2 = 0.1h_1$, $h_3 = 0$.

where the expressions of $a_{ij}(i = 1, \ldots, 5; j = 1, \ldots, 5)$ are shown in Appendix B. The relationship between the wave numbers ξ and wave phase velocity c is determined by searching the condition for non-zero solution of U, V, W, ϕ, T , i.e.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix} = 0. \tag{26}$$

According to any specific wave numbers ξ, the wave phase velocity c can be determined from Eq. (26). Using this method, wave characteristics curves piezoelectric cylindrical laminated shells under large deformation and thermal environment for different response modes are obtained.

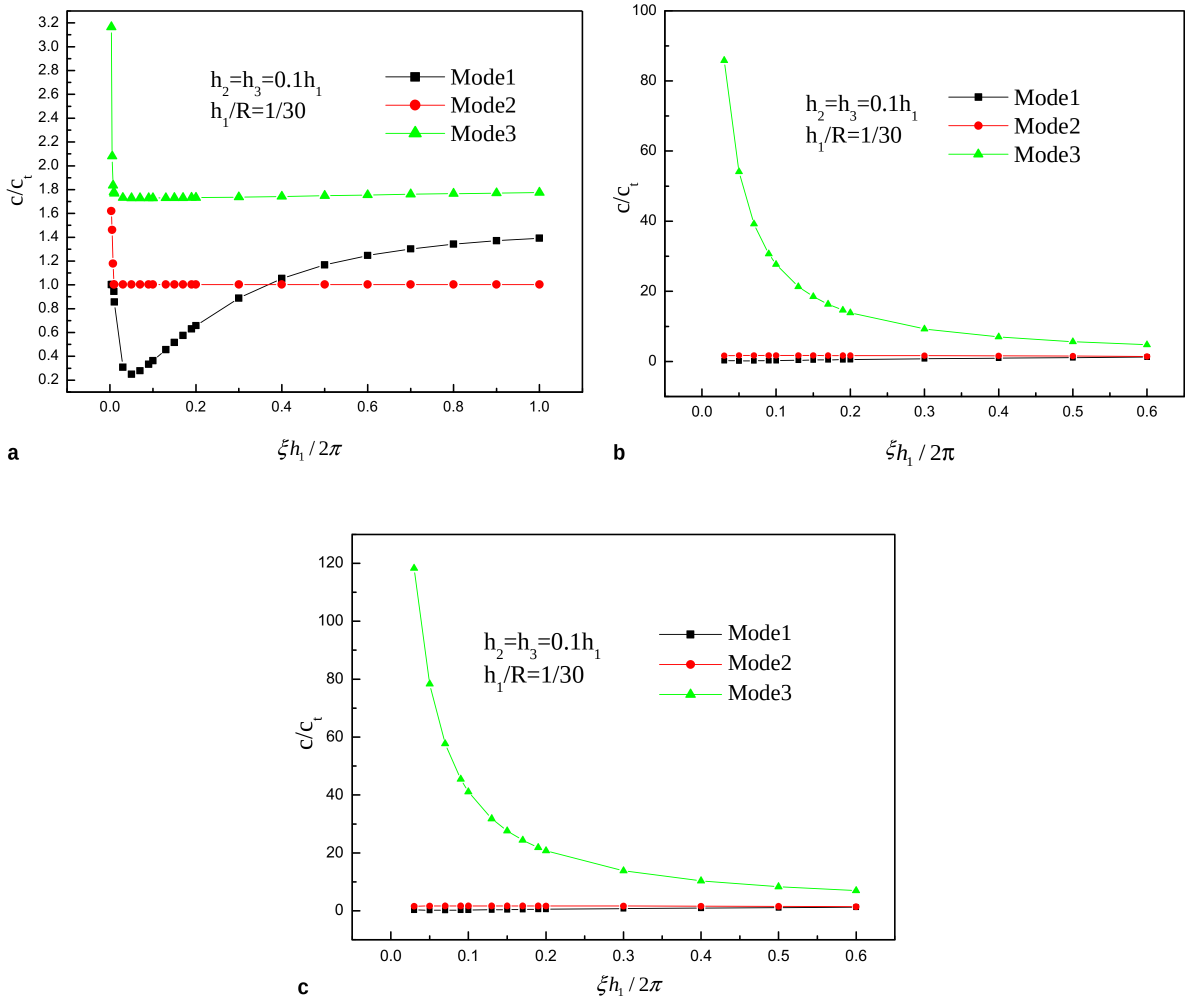

Fig. 3. Wave characteristics of piezoelectric cylindrically laminated shells under large deformation, for the circumferential mode $n = 0$ (a), $n = 2$ (b) and $n = 3$ (c).

4. Numerical results and discussion

Numerical examples are shown in this section. The elastic host layer with the thickness $h_{1,}$ is taken as aluminum to easily compare with a special example in the literature; The PZT-4 material is taken as the outer piezoelectric actuator layer with the thickness, h_2; The PVDF material is taken as the inner piezoelectric sensor layer with the thickness, h_3. All material properties are listed in Appendix C (Li and Lin, 2001; Kadoli and Ganesan, 2004).

To easily investigate the effect of large deformations on wave propagation in piezoelectric cylindrically laminated shells, the non-dimensional wave numbers of wave modes curves is taken as $\xi h/2\pi$ and the non-dimensional velocity is taken as c/c_t (Wang, 2003a,b), where

$$c_t = \left[\frac{c^1_{44p}h_1 + c^2_{44p}h_2 + c^3_{44p}h_3 - c^2_{44p}\frac{(h_1h_2+h_2^2)}{R} + c^3_{44p}\times\frac{(h_1h_3+h_2^2)}{R}}{\rho_1h_1 + \rho_2h_2 + \rho_3h_3}\right]^{1/2}. \tag{27}$$

In order to prove the validity of our solution method and numerical results further, wave propagation in piezoelectric cylindrically laminated shells composed of a piezoelectric layer and a elastic host layer, under small deformation from the present method omitting geometrical non-linear term in geometrical equations and some previous results based on a different analytical method in the literature (Wang, 2003a,b) are shown in Fig. 2. One can see that two results from two different solving methods are nearly agreement.

For $h_1/R = 1/30$ and $h_2 = h_3 = 0.1h_1$, the wave propagation modes at $n = 0$, 2, 3 are described in Fig. 3. It is evidence that wave propagation in a long piezoelectric cylindrically laminated shell with large deformation and rotary inertia, in thermal environment, appear in three different wave modes. For the second and third modes, the non-dimensional phase velocities are two different constant values when the non-dimensional wave number is larger than 0.2, but for the first mode, the non-dimensional phase velocity increases non-linearly as the non-dimensional wave number increases for the circumferential mode $n = 0$ which is shown in Fig. 3(a). Fig. 3(b), (c) and Fig. 4 shows that the circumferential modes have great influence on wave mode, especially for the third wave mode. The Fig. 4 shows that the non-dimensional phase velocity increased with the circumferential mode increased at lower wave numbers, but with the wave numbers increased, the tendency of non-dimensional phase velocity is constant volume for the third wave mode; for the first and second wave mode, the influence of circumferential mode is little.

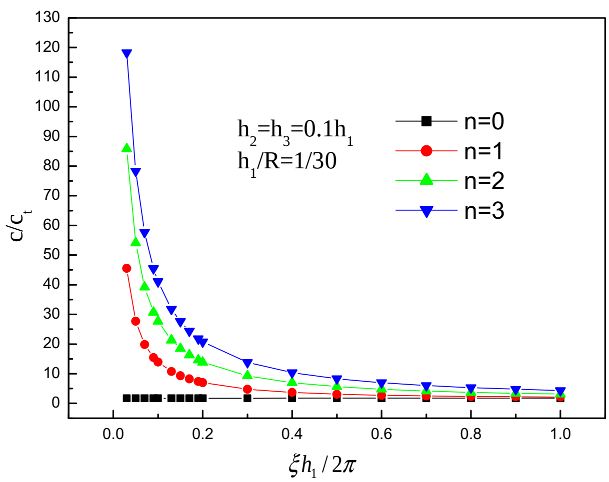

Fig. 4. The third mode of wave characteristics under large deformation for the circumferential mode $n = 0$, $n = 1$, $n = 2$, and $n = 3$.

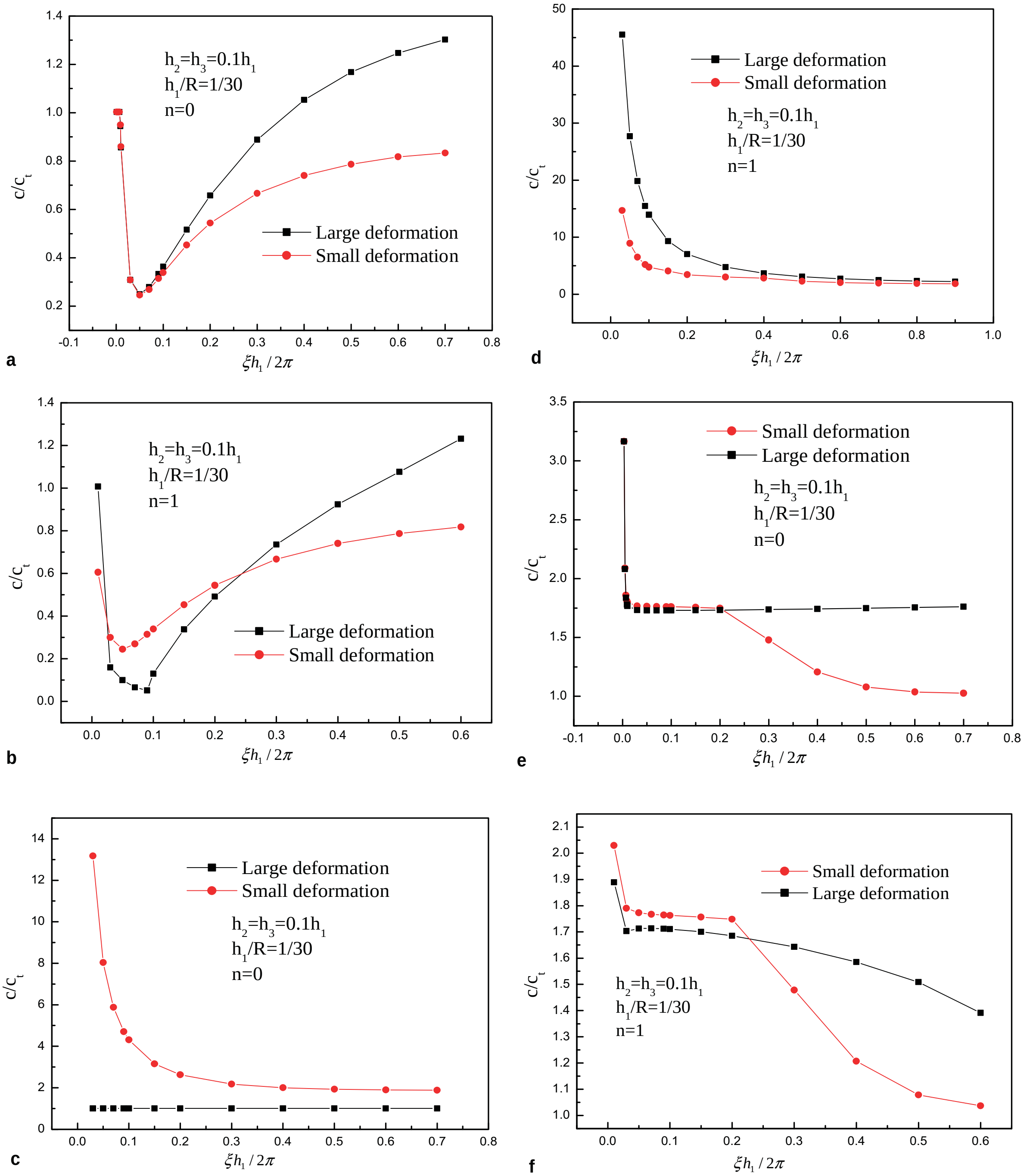

Fig. 5. Wave characteristics of piezoelectric cylindrically laminated shells under large deformation or small deformation, for the first mode at $n = 0$ (a) and $n = 1$ (b), the second mode at $n = 0$ (c), $n = 1$ (d) and the third mode at $n = 0$ (e) and $n = 1$ (f).

The effect of large deformation on wave propagation in piezoelectric cylindrically laminated shells, for the first, second and third wave modes at $n = 0$ and $n = 1$ are plotted in Fig. 5. Fig. 5(a) and (b) shows the comparison of the wave phase velocity under large deformation (geometrical non-linear) with the wave phase velocity under small deformation (geometrical linear) for the first mode at $n = 0$ and $n = 1$. It is seen that the phase velocity decreases dramatically within a very small range of wave numbers at first, and it increase smoothly with higher value. The large deformation of piezoelectric cylindrically laminated shells has only a little effect on the phase velocity at lower wave number near zero, but it has dramatic influence as wave numbers increases, and the effect of large deformation on wave propagation in the piezoelectric cylindrically laminated shells is dependent on the circumferential number n of the piezoelectric cylindrically laminated shells. The wave phase velocity of the second mode for $n = 0$ and $n = 1$ are shown in Fig. 5(c) and (d). It is seen that the wave phase velocity under larger deformation is evidently different from that under small deformation at lower wave numbers, but the wave velocities for $n = 1$, under two different deformation conditions converge gradually two constant values as the wave number increases. Fig. 5(e) and (f) show the wave phase velocity of the third mode for $n = 0$ and $n = 1$. It is seen from Fig. 5(e) that the phase velocity for $n = 0$, under larger deformation appear in a constants as wave numbers increase, but the phase velocity for $n = 0$, under small deformation decreases as wave numbers increase. It is seen from Fig. 5(f) that the phase velocity for $n = 1$, under larger deformation is lower than that under small deformation when the wave number is less than 0.2, but the phase velocity for $n = 1$, under larger deformation is much larger than that under small deformation when the wave number is larger than 0.2.

The effect of rotary inertia on wave propagation, for the first, second and third modes at $n = 0$, in piezoelectric cylindrically laminated shells is described in Fig. 6. The result shows that the rotary inertia has very little effect on wave propagation for the second and third modes, in the piezoelectric cylindrically laminated shells. But as the wave number increases, the phase velocity considering rotary inertia for the first mode is much lower than the phase velocity no considering rotary inertia.

Fig. 7 shows the effect of the thermal load on wave propagation for the first mode and third mode at $n = 0$ and $n = 1$ in piezoelectric cylindrically laminated shells. It is seen that for $n = 0$, the effect of thermal load on wave propagation in the piezoelectric cylindrically laminated shells is very little for the first mode; But, for $n = 1$, when the wave number is larger than 0.3, the magnitude of phase velocity considering thermal loading is lower than that no considering thermal loading for the first mode. For the third mode, the effect of thermal load is very little.

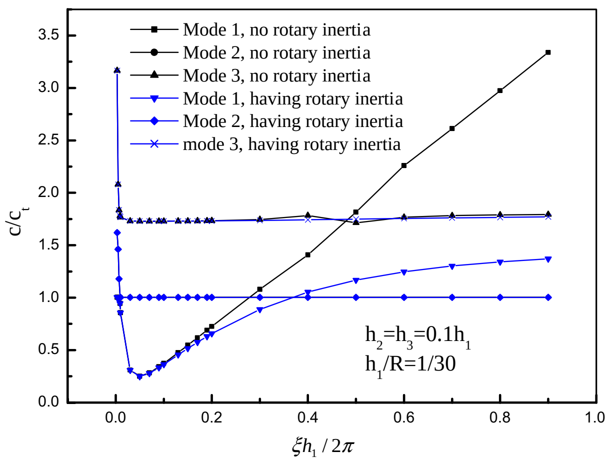

Fig. 6. The effect of rotary inertia on wave characteristics of piezoelectric cylindrically laminated shells, at the circumferential mode $n = 0$.

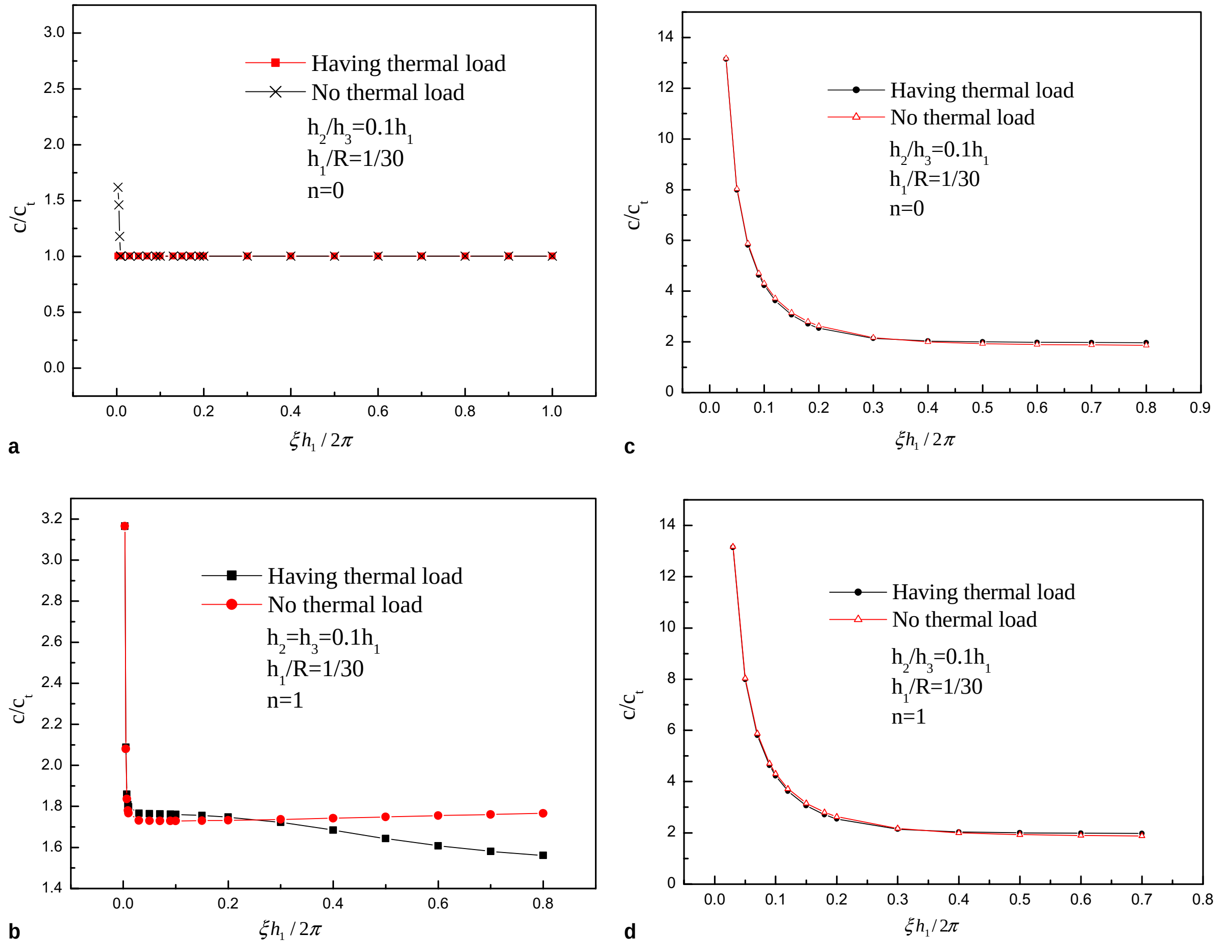

Fig. 7. The effect of thermal load on wave characteristics of piezoelectric cylindrically laminated shells, for the first mode at $n = 0$ (a) and $n = 1$ (b), the third mode at $n = 0$ (c) and $n = 1$ (d).

5. Conclusions

The main contribution in this paper is to describe the effects of large deformation, rotary inertia and thermal load on wave propagation in a piezoelectric cylindrically laminated shell. Utilizing Hamilton's principle, nonlinear dynamic governing equations of the piezoelectric cylindrically laminated shells are derived. The wave propagation mode curves are obtained by solving an eigenvalue problem. From results carried out some conclusions are obtained by

(1) The effect of large deformation on wave propagation in piezoelectric cylindrically laminated shells is dependent on the wave modes, wave number and circumferential modes. The large deformation has little effect on the phase velocity at lower wave numbers, but it have evidence effect with the increasing of wave numbers for the first and third wave modes. For the second wave mode, the deformation has evidence effect at lower wave numbers, but it has little evidence on higher wave numbers. The circumferential modes have great influence for the third wave mode at lower wave number.

(2) Rotary inertia has very little effect on wave propagation, for the second and third wave modes, in piezoelectric cylindrically laminated shells; as the wave number increases, the phase velocity considering rotary inertia for the first mode is much lower than the phase velocity no considering rotary inertia.
(3) The effect of the thermal load on wave propagation in piezoelectric cylindrically laminated shells is mainly dependent on the circumferential modes of piezoelectric cylindrically laminated shells. The effect of thermal load on wave propagation in the piezoelectric cylindrically laminated shells for the first mode at $n = 0$ is very little. But, for $n = 1$, when the wave number is larger than 0.3, the magnitude of phase velocity considering thermal loading is lower than that no considering thermal load. For the third wave mode, the effect of thermal load is little.

The solution method in the paper may be used as a useful reference to investigate wave propagation in piezoelectric cylindrically laminated shells under large deformation and rotary inertia, in thermal environment, for various laminated materials, the layers numbers and thickness of piezoelectric laminated shells. The aim of comprehending the characteristic of wave propagation in piezoelectric cylindrically laminated shells under large deformation and rotary inertia is used to predict the wave response in the higher frequencies range so that the smaller size of damage in this piezoelectric laminated structures can be detected with good resolution. The results carried out can be used in the ultrasonic inspection techniques and structural health monitoring.

Acknowledgement

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Appendix A

$$A_1 = c^1_{33p}h_1 + c^2_{33p}h_2 + c^3_{33p}h_3, \quad A_2 = c^1_{13p}h_1 + c^2_{13p}h_2 + c^3_{13p}h_3,$$

$$A_3 = \frac{1}{2}c^2_{33p}(h_2^2 + h_1h_2) - \frac{1}{2}c^3_{33p}(h_3^2 + h_1h_3), \quad A_4 = \frac{1}{2}c^2_{13p}(h_2^2 + h_1h_2) - \frac{1}{2}c^3_{13p}(h_3^2 + h_1h_3),$$

$$A_5 = e^2_{33p}h_2 + e^3_{33p}h_3, \quad A_6 = -\lambda^1_2h_1 - \lambda^2_2h_2 - \lambda^3_2h_3, \quad B_1 = c^1_{13p}h_1 + c^2_{13p}h_2 + c^3_{13p}h_3,$$

$$B_2 = c^1_{11p}h_1 + c^2_{11p}h_2 + c^3_{11p}h_3,$$

$$B_3 = \frac{1}{2}c^2_{13p}(h_2^2 + h_1h_2) - \frac{1}{2}c^3_{13p}(h_3^2 + h_1h_3), \quad B_4 = \frac{1}{2}c^2_{11p}(h_2^2 + h_1h_2) - \frac{1}{2}c^3_{11p}(h_3^2 + h_1h_3),$$

$$B_5 = e^2_{31p}h_2 + e^3_{31p}h_3, \quad B_6 = -\lambda^2_1h_1 - \lambda^2_1h_2 - \lambda^3_1h_3, \quad C_1 = c^1_{44p}h_1 + c^2_{44p}h_2 + c^3_{44p}h_3,$$

$$C_2 = \frac{1}{2}c^2_{44p}(h_2^2 + h_1h_2) - \frac{1}{2}c^3_{44p}(h_3^2 + h_1h_3), \quad C_3 = e^1_{15p}h_1 + e^2_{15p}h_2,$$

$$D_1 = \frac{1}{2}c^2_{33p}(h_2^2 + h_1h_2) - \frac{1}{2}c^3_{33p}(h_3^2 + h_1h_3), \quad D_2 = \frac{1}{2}c^2_{13p}(h_2^2 + h_1h_2) - \frac{1}{2}c^3_{13p}(h_3^2 + h_1h_3),$$

$$D_3 = \frac{1}{12}c^1_{33p}h_1^3 + \frac{c^2_{33p}}{3}(h_2^3 + \frac{3}{2}h_1h_2^2 + \frac{3}{4}h_1^2h_2) + \frac{c^3_{33p}}{3}(h_3^3 + \frac{3}{2}h_1h_3^2 + \frac{3}{4}h_1^2h_3),$$

$$D_4 = \frac{1}{12}c^1_{13p}h_1^3 + \frac{c^2_{13p}}{3}(h_2^3 + \frac{3}{2}h_1h_2^2 + \frac{3}{4}h_1^2h_2) + \frac{c^3_{13p}}{3}(h_3^3 + \frac{3}{2}h_1h_3^2 + \frac{3}{4}h_1^2h_3),$$

$$D_5 = \frac{e_{33P}^2}{2}(h_1h_2 + h_2^2) - \frac{e_{33p}^3}{2}(h_1h_3 + h_3^2), \quad D_6 = -\frac{\lambda_2^2}{2}(h_1h_2 + h_2^2) + \frac{\lambda_2^3}{2}(h_1h_3 + h_3^2),$$
$$E_1 = \frac{1}{2}c_{13p}^2(h_2^2 + h_1h_2) - \frac{1}{2}c_{13p}^3(h_3^2 + h_1h_3), \quad E_2 = \frac{1}{2}c_{11p}^2(h_2^2 + h_1h_2) - \frac{1}{2}c_{11p}^3(h_3^2 + h_1h_3),$$
$$E_3 = \frac{1}{12}c_{13p}^1h_1^3 + \frac{c_{13p}^2}{3}(h_2^3 + \frac{3}{2}h_1h_2^2 + \frac{3}{4}h_1^2h_2) + \frac{c_{13p}^3}{3}(h_3^3 + \frac{3}{2}h_1h_3^2 + \frac{3}{4}h_1^2h_3),$$
$$E_4 = \frac{1}{12}c_{11p}^1h_1^3 + \frac{c_{11p}^2}{3}(h_2^3 + \frac{3}{2}h_1h_2^2 + \frac{3}{4}h_1^2h_2) + \frac{c_{11p}^3}{3}(h_3^3 + \frac{3}{2}h_1h_3^2 + \frac{3}{4}h_1^2h_3),$$
$$E_5 = \frac{e_{31P}^2}{2}(h_1h_2 + h_2^2) - \frac{e_{31p}^3}{2}(h_1h_3 + h_3^2), \quad E_6 = -\frac{\lambda_1^2}{2}(h_1h_2 + h_2^2) + \frac{\lambda_1^3}{2}(h_1h_3 + h_3^2),$$
$$F_1 = \frac{1}{2}c_{44p}^2(h_2^2 + h_1h_2) - \frac{1}{2}c_{44p}^3(h_3^2 + h_1h_3),$$
$$F_2 = \frac{1}{12}c_{44p}^1h_1^3 + \frac{c_{44p}^2}{3}(h_2^3 + \frac{3}{2}h_1h_2^2 + \frac{3}{4}h_1^2h_2) + \frac{c_{44p}^3}{3}(h_3^3 + \frac{3}{2}h_1h_3^2 + \frac{3}{4}h_1^2h_3),$$
$$F_3 = \frac{e_{15P}^2}{2}(h_1h_2 + h_2^2) - \frac{e_{15p}^3}{2}(h_1h_3 + h_3^2),$$

where $c_{11p}^i,\ c_{12p}^i,\ c_{22p}^i,\ c_{44p}^i,\ e_{11p}^i,\ e_{12p}^i,\ e_{24p}^i,\ e_{15p}^i$ and $g_{11p}^i,\ g_{22p}^i,\ g_{33p}^i$ are given by

$$c_{11p}^i = c_{11}^i - \frac{c_{13}^ic_{13}^i}{c_{33}^i}, \quad c_{12p}^i = c_{12}^i - \frac{c_{13}^ic_{23}^i}{c_{33}^i}, \quad c_{22p}^i = c_{22}^i - \frac{c_{23}^ic_{23}^i}{c_{33}^i}, \quad c_{44p}^i = c_{44}^i,$$
$$e_{31p}^i = e_{31}^i - \frac{c_{13}^ie_{33}^i}{c_{33}^i}, \quad e_{32p}^i = e_{32}^i - \frac{c_{23}^ie_{33}^i}{c_{33}^i}, \quad e_{24p}^i = e_{24}^i, \quad e_{15p}^i = e_{15}^i,$$
$$g_{11p}^i = g_{11}^i + \frac{e_{33}^ie_{33}^i}{c_{33}^i}, \quad g_{22p}^i = g_{22}^i, \quad g_{33p}^i = g_{33}^i, \quad i = 1, 2, 3 \text{ (Layer number)}.$$

Appendix B

$$a_{11} = -A_1\xi^2 - \frac{C_1}{R^2}n^2 + (\rho_1h_1 + \rho_2h_2 + \rho_3h_3)\xi^2c^2, \quad a_{12} = \frac{A_2}{R}i\xi n + \frac{A_4}{R^2}i\xi n + \frac{C_1}{R}i\xi n + \frac{C_2}{R^2}i\xi n,$$
$$a_{13} = \frac{A_2}{R}i\xi + A_3i\xi^3 + \frac{A_4}{R^2}i\xi n^2 + 2\frac{C_2}{R^2}i\xi n^2 - \frac{I}{R}\xi^3c^2i, \quad a_{14} = -A_5\xi^2 - \frac{C_3}{R^2}n^2, \quad a_{15} = A_6i\xi,$$
$$a_{21} = -\left(\frac{B_1}{R} + \frac{C_1}{R} + \frac{E_1}{R^2} + \frac{F_1}{R^2}\right)i\xi n,$$
$$a_{22} = -\left(\frac{B_2}{R^2} + \frac{B_4}{R^3} + \frac{E_2}{R^3} + \frac{E_4}{R^4}\right)n^2 - (C_1 + \frac{C_2}{R} + \frac{F_1}{R} + \frac{F_2}{R^2})\xi^2 + (\rho_1h_1 + \rho_2h_2 + \rho_3h_3)\xi^2c^2 - \frac{I}{R^2}\xi^2c^2,$$
$$a_{23} = -\left(\frac{B_2}{R} + \frac{E_2}{R^3}\right)n + \left(\frac{B_3}{R} + \frac{2C_2}{R} + \frac{2F_2}{R^2} + \frac{E_3}{R^2}\right)\xi^2n - \left(\frac{B_4}{R^3} + \frac{E_4}{R^3}\right)n^3 + \frac{I}{R^2}n\xi^2c^2,$$
$$a_{24} = -\left(\frac{B_5}{R} + \frac{C_3}{R} + \frac{E_5}{R^2} + \frac{F_2}{R^2}\right)i\xi n, \quad a_{25} = -\left(\frac{B_6}{R} + \frac{E_6}{R^2}\right)n,$$
$$a_{31} = -D_1\xi^3i - \left(\frac{F_1}{R^2} + \frac{E_1}{R^2} + \frac{F_1}{R^2}\right)i\xi n^2 - \frac{B_1}{R}i\xi + \frac{I}{R}\xi^3c^2i,$$

$$a_{32} = -\left(\frac{D_2}{R} + \frac{D_4}{R^2} + 2\frac{F_1}{R} + 2\frac{F_2}{R^2}\right)\xi^2 n - \left(\frac{E_2}{R^3} + \frac{E_4}{R^4}\right)n^3 + \left(-\frac{B_2}{R^2} - \frac{B_4}{R^3}\right) + \frac{2I}{R^2}n\xi^2 c^2,$$

$$a_{33} = -\left(\frac{D_2}{R} - \frac{B_3}{R}\right)\xi^2 - D_3\xi^4 - \left(\frac{D_4}{R^2} + \frac{2F_2}{R^2} + \frac{E_3}{R^2} + \frac{2F_2}{R^2}\right)n^2\xi^2 - \left(\frac{E_2}{R^3} + \frac{B_4}{R^3}\right)n^2$$
$$-\frac{E_4}{R^4}n^4 - \frac{B_2}{R^2} + (\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3)\xi^2 c^2 - I\xi^4 c^2 + \frac{I}{R^2}\xi^2 n^2 c^2,$$

$$a_{34} = -D_5\xi^3 i - \frac{F_3}{R^3}i\xi n^2 - \left(\frac{E_5}{R^2} + 2\frac{F_3}{R^2}\right)i\xi n^2 - \frac{B_5}{R}i\xi,$$

$$a_{35} = -D_6\xi^2 - \frac{E_6}{R^2}n^2 - \frac{B_6}{R}, \quad a_{41} = -e^2_{33p}h_2 - \frac{e^2_{15p}}{R^2}h_2 n^2,$$

$$a_{42} = \frac{e^2_{31p}}{R}h_2\xi ni + \frac{e^2_{31p}}{2R^2}(h_1h_2 + h_2^2)\xi ni + \frac{e^2_{15p}}{R}h_2\xi ni + \frac{e^2_{15p}}{2R^2}(h_1h_2 + h_2^2)\xi ni,$$

$$a_{43} = -\frac{1}{2}e^2_{33p}(h_1h_2 + h_2^2)\xi^3 i + \frac{e^2_{31p}}{R}h_2\xi i + \frac{e^2_{31p}}{2R^2}(h_1h_2 + h_2^2)\xi n^2 i + \frac{e^2_{15p}}{R^2}(h_1h_2 + h_2^2)2\xi n^2,$$

$$a_{44} = g^2_{33p}h_2\xi^2 + g^2_{11p}h_2\frac{n^2}{R^2}, \quad a_{45} = p_3^2 h_2, a_{51} = -e^3_{33p}h_3 - \frac{e^3_{15p}}{R^2}h_3 n^2,$$

$$a_{52} = \frac{e^3_{31p}}{R}h_3\xi ni + \frac{e^3_{31p}}{2R^2}(h_1h_3 + h_3^2)\xi ni + \frac{e^3_{15p}}{R}h_3\xi ni + \frac{e^3_{15p}}{2R^2}(h_1h_3 + h_3^2)\xi ni,$$

$$a_{53} = -\frac{1}{2}e^3_{33p}(h_1h_3 + h_3^2)\xi^3 i + \frac{e^3_{31p}}{R}h_2\xi i + \frac{e^3_{31p}}{2R^2}(h_1h_3 + h_2^3)\xi n^2 i + \frac{e^3_{15p}}{R^2}(h_1h_3 + h_3^2)\xi n^2,$$

$$a_{54} = g^3_{33p}h_3\xi^2 + g^3_{11p}h_3\frac{n^2}{R^2}, a_{55} = p_3^3 h_2.$$

Appendix C

Aluminum: $\rho_1 = 2.8 \times 10^3$ kg/m^3, $c^1_{11} = c^1_{22} = c^1_{33} = 105\ GPa, c^1_{12} = c^1_{13} = 51\ GPa$, Coefficient of thermal expansion $\alpha^1 = 2.55 \times 10^{-5}/°C$ PZT-4: $\rho_2 = 7.5 \times 10^3 \text{kg/m}^3, c^2_{11} = c^2_{22} = 132$ GPa, $c^2_{12} = 71$ GPa, $c^2_{13} = 73$ GPa, $c^2_{33} = 115$ GPa, $c^2_{66} = 26$ GPa; Piezoelectric properties (k/m^2): $e^2_{31} = -4.1, e^2_{33} = 14.1, e^2_{15} = 10.5$; Dielectric constant ($\phi$/m): $g^2_{11} = 5.841 \times 10^{-9}, g^2_{33} = 7.124 \times 10^{-9}$, Coefficient of thermal expansion $\alpha^2_{11} = \alpha^2_{22} = 1.2 \times 10^{-6}/°C$, Pyroelectric constant $p^2_3 = 0.25 \times 10^{-4}$ PVDF: $\rho_3 = 1.8 \times 10^3$ kg/m^3, $c^3_{11} = 3.61$ GPa, $c^3_{12} = 1.61$ GPa, $c^3_{13} = 1.42$ GPa, $c^3_{22} = 3.13$ GPa, $c^3_{33} = 1.63$ GPa, $c^3_{66} = 0.69$ GPa; Piezoelectric properties (k/m^2): $e^3_{31} = 32.075 \times 10^{-3}, e^3_{33} = -21.19 \times 10^{-3}, e^3_{15} = -15.93 \times 10^{-3}$; Dielectric constant ($\phi$/m): $g^3_{11} = 53.985 \times 10^{-12}, g^3_{33} = 59.295 \times 10^{-12}$, Coefficient of thermal expansion $\alpha^3_{11} = \alpha^3_{22} = 1.2 \times 10^{-4}/°C$, Pyroelectric constant $p^3_3 = -4 \times 10^{-5}$.

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